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ELASTIC ANALYSIS OF ROTATING SPHERICAL PRESSURE VESSEL OF FUNCTIONALLY GRADED MATERIAL MODELED BY MORI-TANAKA SCHEME

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Abstract

The present study aims to elastic analysis of rotating spherical pressure vessels made of functionally graded materials (FGMs), by finite element method based on principle of stationary total potential (PSTP). Material properties are graded along radial direction, using Mori-Tanaka scheme. The effects of gradation of material properties on stress, strain and deformation behavior of the shell are investigated and a comparison of stresses, strains and deformations for different grading parameters in metal-ceramic FGM shell is presented. Results obtained shows that there is a significant reduction in stresses and deformation behavior of the FGM shells as compared to homogeneous shell. Further it is observed that metal-ceramic FGM shell having value of grading parameter $n = 2$, has the lowest overall stresses and therefore can be most efficiently employed for rotating spherical pressure vessel.

Keywords *:* Functionally graded material (FGM), Linear elastic analysis, Rotating spherical shell, Finite element method (FEM).

1. INTRODUCTION

Functionally graded materials (FGMs) are special composite materials that have continuous and smooth spatial variations of physical and mechanical properties. Gradation of the material properties in FGMs is achieved by varying the volume fractions of the constituents. Functionally graded shells are widely used in space vehicles, aircrafts, nuclear power plants and many other engineering applications [1]. Control and optimization of total stresses and displacement fields due to internal pressure and centrifugal force is an important task, which is achieved by varying the material property in FGM pressure vessels.

Many researchers have worked on elastic analysis of rotating spherical shells, conical shells, cylindrical shells, disks etc. by analytical as well as approximate methods such as finite element method. N. Tutuncu, M. Ozturk [2] reported closed form solution for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure, using the infinitesimal theory of elasticity. The material stiffness obeying a power law is assumed to vary through the wall thickness and poisson's ratio is assumed constant. F. Alavi, D. Karimi and A. Bagri [3] studied the thermoelastic behavior of thick functionally graded hollow sphere under thermal and mechanical loads. The mechanical and thermal properties of FG sphere are assumed to be functions of radial position. Two methods were used to estimate the effective mechanical properties of FG sphere. One is the Rule of Mixture (R-M) scheme and another is Mori-Tanaka scheme. N. Tutunku and B. Temel [4] analyzed axisymmetric displacements and stresses in functionally-graded hollow cylinders, disks and spheres subjected to uniform internal pressure using plane elasticity theory and complementary functions method. The material is assumed to be functionally graded in the radial direction. Variations in the material properties such as Young's modulus and Poisson's ratio are assumed to be arbitrary functions of the radial coordinate. This assumption yields a two-point boundary value problem with a governing differential equation of variable coefficients which are solved by complimentary function method. Y. Z. Chen and X. Y. Lin [5] presented an alternative solution for the problems of thick-walled cylinders and spheres of functionally graded materials (FGMs). Displacement function is chosen as unknown in the governing equation. The Young's modulus is arbitrary and constant Poisson's ratio is assumed. Two fundamental solutions are defined from a

numerical solution under two particular initial boundary conditions. In addition, the transmission matrix M is defined for the single layer as well as multi-layered shells. The matrix relates the values of radical stress and displacement at the initial point to those at the end point of the layer.

In recent work V.T.T. Anh, D.H. Bich and N.D. Duc [6] dealt with nonlinear stability analysis of thin annular spherical shells made of functionally graded materials (FGM) on elastic foundations under external pressure and temperature. Material properties are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of constituents. V.R. Kar and S.K. Panda [7] examined the linear and the nonlinear deformation behaviour of functionally graded (FG) spherical shell panel under thermo-mechanical load. The temperature dependent effective material properties of FG shell panel are evaluated using Voigt's micro-mechanical rule in conjunction with power-law distribution. H.V. Tung [8] presented an analytical approach to investigate the nonlinear axisymmetric response of Functionally Graded (FG) Shallow Spherical Shells (SSSs) resting on elastic foundations, exposed to thermal environment and subjected to uniform external pressure. Material properties are assumed to be temperature-independent and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of constituents.

In present research work rotating spherical pressure vessels subjected to hydrostatic internal pressure are analyzed. Governing equations are derived by finite element method based on principle of stationary total potential. The shells are made of functionally graded material of aluminum metal and zirconia ceramic. Functionally gradation of the material properties is done by Mori-Tanaka scheme. The work aims at investigating the effects of functionally gradation of the material properties on the deformation stresses and strains of the shells for metal-ceramic FGM. At the same time the effects of grading parameter is also find out and presented in the form of graph for some numerical problem.

2. MATERIAL MODELING:

The effective bulk modulus and shear modulus of the FGM disk, evaluated using the Mori-Tanaka scheme [9] are given by:

$$B(r) = (B_o - B_i)/V_o\left(1+(1-V_o)\frac{3(B_o - B_i)}{3B_i + 4G_i}\right) + B_i \quad (1)$$

$$G(r) = (G_o - G_i)/V_o\left(1+(1-V_o)\frac{(G_o - G_i)}{G_i + f_1}\right) + G_i \quad (2)$$

$$f_1 = \frac{G_i(9B_i + 8G_i)}{6(B_i + 2G_i)} \quad (3)$$

Here, V is the volume fraction of the phase material. The subscripts i and o refer to the inner and outer materials respectively. The volume fraction of the inner and outer phases are related by

$$V_i + V_o = 1 \quad (4)$$

And V_o is expressed as

$$V_o = \left(\frac{r - r_i}{r_o - r_i}\right)^n \quad (5)$$

Where $n\ (n \geq 0)$ is the volume fraction exponent. The elastic modulus E can be found as

$$E(r) = \frac{9B(r)G(r)}{3B(r) + G(r)} \quad (6)$$

The mass density ρ can be given by the rule of mixtures as:

$$\rho(r) = (\rho_o - \rho_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n + \rho_i \quad (7)$$

3. FINITE ELEMENT MODELING :

For axisymmetric problem, an (*r*-*z*) plane (analogous to *x*-*y* plane for plane elasticity) can be considered. Four independent nonzero strains exist in axisymmetric problems [10].

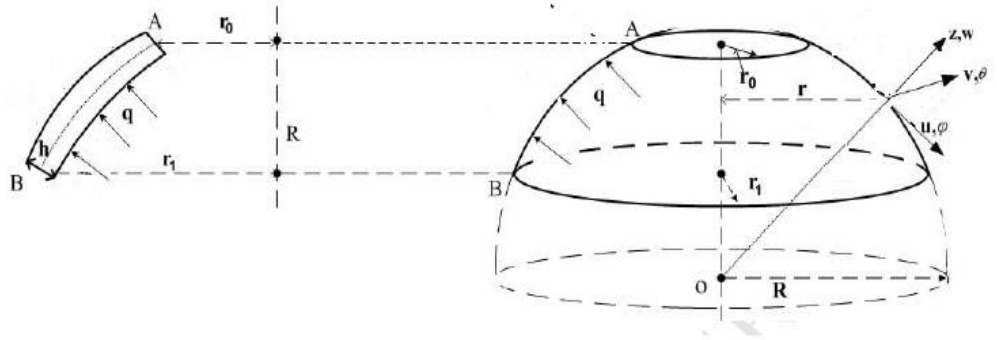


Figure 1: Geometry of spherical shell

$$\varepsilon_r = \frac{\partial u}{\partial r}\ , \qquad \varepsilon_\theta = \frac{u}{r}\ , \qquad \varepsilon_z = \frac{\partial v}{\partial z}, \qquad \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \tag{1}$$

In standard finite element notation, strain displacement relationship can be written as:

$$\{\varepsilon\} = [B]\{\delta\}^e \tag{9}$$

Where [*B*] is strain displacement relation matrix which depends on element taken and contains derivatives of shape functions.

From generalized hooks law, components of total strain in radial, circumferential and axial direction is given by

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\sigma_\theta - \nu\sigma_z\right) \tag{10}$$

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_r - \nu\sigma_z\right) \tag{11}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_\theta - \nu\sigma_r\right) \tag{12}$$

By solving above three equations, stress strain relationship can be obtained as follows:

$$\sigma_r = \frac{E(r)}{(1-2\nu)(1+2\nu)}\left[(1-\nu)\varepsilon_r + \nu\varepsilon_z + \nu\varepsilon_\theta\right] \tag{13}$$

Similarly axial and circumferential stress can also be obtained.

In standard finite element matrix notation above stress strain relations can be written as:

$$\{\sigma\} = [D(r)]\left(\{\varepsilon\} - \{\varepsilon\}^0\right) + \{\sigma\}^0 \tag{14}$$

Where

$$\{\sigma\} = \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} \tag{15}$$

$$D(r) = \frac{(1-\nu)E(r)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{(1-\nu)} & \frac{\nu}{(1-\nu)} & 0 \\ \frac{\nu}{(1-\nu)} & 1 & \frac{\nu}{(1-\nu)} & 0 \\ \frac{\nu}{(1-\nu)} & \frac{\nu}{(1-\nu)} & 1 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{16}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_r \\ \partial_\theta \\ \varepsilon_z \\ \gamma_{rz} \end{Bmatrix} \tag{17}$$

Where $D(r)$ is a function of radius r and $\{\sigma\}^0$ and $\{\varepsilon\}^0$ are initial stress and strain. Taking initial stress and strain zero, equation (14) reduces to

$$\{\sigma\} = [D(r)]\{\varepsilon\} \tag{18}$$

When the shell rotates and is subjected to internal pressure, it experiences a distributed force over its volume and surface. Under these forces when shell is properly supported (so as to prevent rigid body motion), it undergoes deformation and stores internal strain energy *U*, which is given by:

$$U = \frac{1}{2}\int_v \{\varepsilon\}^T \{\sigma\}\ dv \tag{19}$$

Also the potential of external body and surface force is given by

$$V = -\int_v \{\delta\}^T \{q_v\}\, dv - \int_s \{\delta\}^T \{q_s\}\, ds \tag{20}$$

The element level equation can be written as

$$U^e = \int_v \frac{1}{2}\{\delta\}^{eT} [B]^T [D(r)][B]\{\delta\}^e\ dv \tag{21}$$

$$V^e = -\int_v \{\delta\}^{eT} [N]^T \{q_v\}\, dv - \int_s \{\delta\}^{eT} [N]^T \{q_s\}\, ds \tag{22}$$

The total potential of the element can be written as

$$\pi_p{}^e = \int_v \frac{1}{2}\{\delta\}^{eT} [B]^T [D(r)][B]\{\delta\}^e\ dv - \int_v \{\delta\}^{eT} [N]^T \{q_v\}\, dv - \int_s \{\delta\}^{eT} [N]^T \{q_s\}\, ds \tag{23}$$

Defining element stiffness matrix $[K]^e$ and element load vector $\{f\}^e$ as:

$$[K]^e = \int_v [B]^T [D(r)][B]\ dv \tag{24}$$

$$\{f\}^e = \int_v [N]^T \{q_v\}\, dv + \int_s [N]^T \{q_s\}\, ds \tag{25}$$

Total potential energy of the shell is given by

$$\pi_p = \Sigma \pi_p{}^e \tag{26}$$

Using the Principle of stationary total potential (PSTP), the total potential is set to be stationary with respect to small variation in the nodal degree of freedom that is:

$$\frac{\partial \pi_p}{\partial \{\delta\}^T} = 0 \tag{27}$$

which gives system level equation for shell as:

$$[K]\{\delta\} = \{F\} \tag{28}$$

where

$$[K] = \sum_{n=1}^{N} [K]^e = Globel\ Stiffness\ matrix \tag{29}$$

$$\{F\} = \sum_{n=1}^{N} [f]^e = Globel\ load\ vector \tag{30}$$

N = no. of elements.

The summation indicates assembly of individual elemental matrices following the standard procedure of assembly.

4. RESULTS AND DISCUSSION :

A. Validation: To validate the current work, problems of reference [4] are reconsidered and two types of material models are analyzed. Following equations are used for material modelling in reference [4]**:**

$$V_c = \left(\frac{x-1}{k-1}\right)^n \tag{31}$$

$$\nu = \nu_c V_c + \nu_m (1 - V_c) \tag{32}$$

$$E = E_c V_c + E_m (1 - V_c) \tag{33}$$

where V is the volume fraction, E is young's modulus and v is poisson's ratio, subscript 'm' refers to metal and 'c' refers to ceramic. x is non-dimensional radius that is r/a and k is the ratio of outer diameter to inner diameter.

In model 1, poisson's ratio is taken constant (0.333) and young's modulus vary according to equation (33) taking n = 1 while in model 2, both poisson's ratio and young's modulus vary according to equation (32) and equation (33) taking $n = k = 2$. Material properties of the metal and ceramic used are: E_m = 200 *GPa*, E_c = 360 *GPa*, ν_m = 0.333, ν_c = 0.2 and the vessels are subjected to unit internal pressure that is 1 *GPa*.

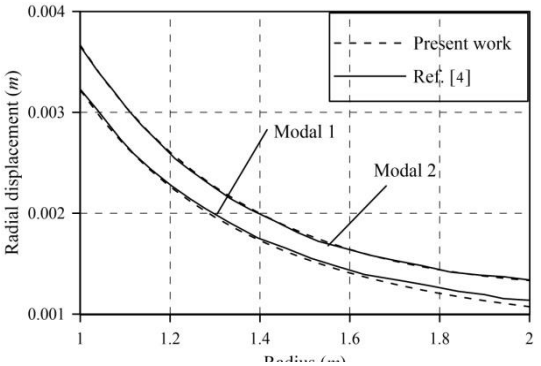


Figure 2: Comparison of radial displacement with Ref. [4]

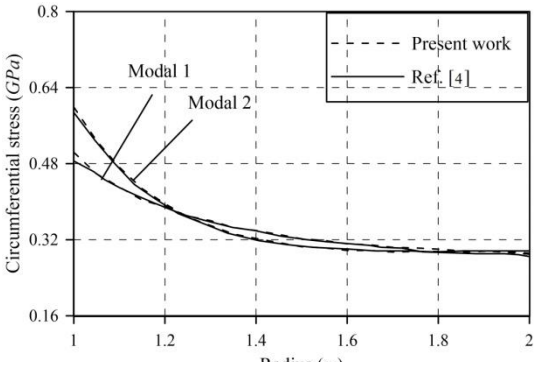


Figure 3: Comparison of circumferential stress with Ref. [4]

Figure 2 and Figure 3 shows the comparison of radial displacement and circumferential stress with the results of reference [4]. It can be seen that both the results are in good agreement.

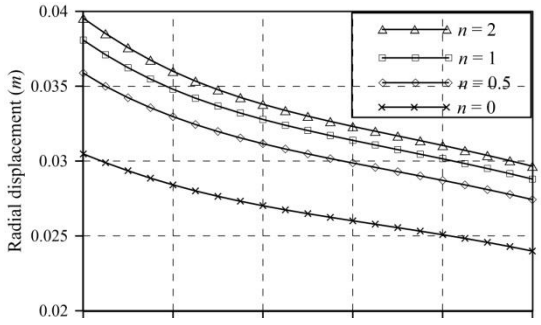


Figure 4: Distribution of radial displacement

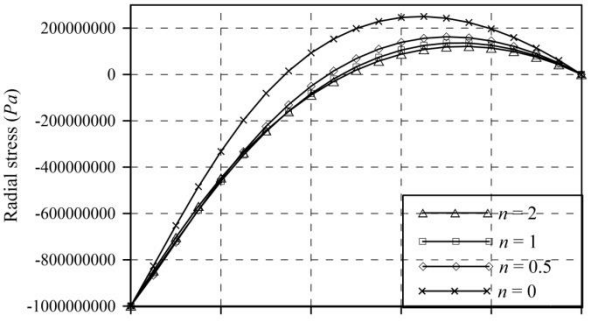


Figure 7: Distribution of radial stress

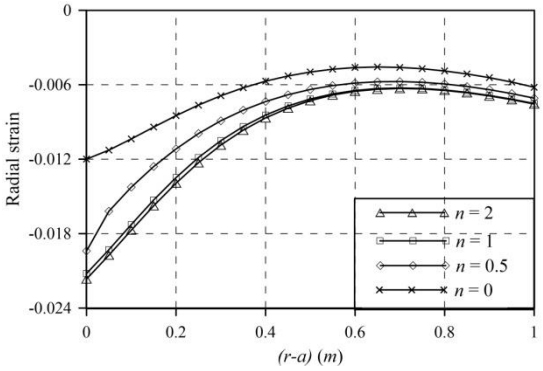


Figure 5: Distribution of radial strain

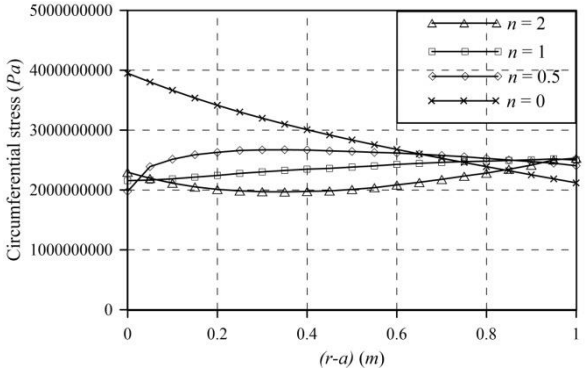


Figure 8: Distribution of circumferential stress

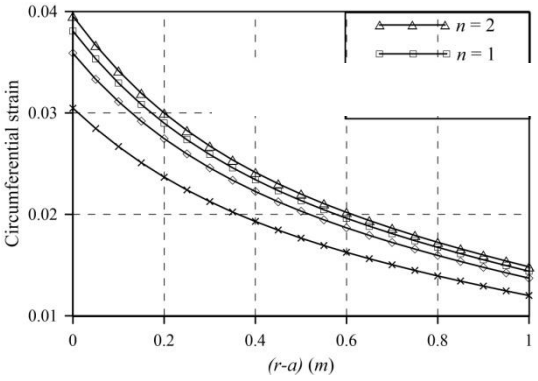


Figure 6: Distribution of circumferential strain

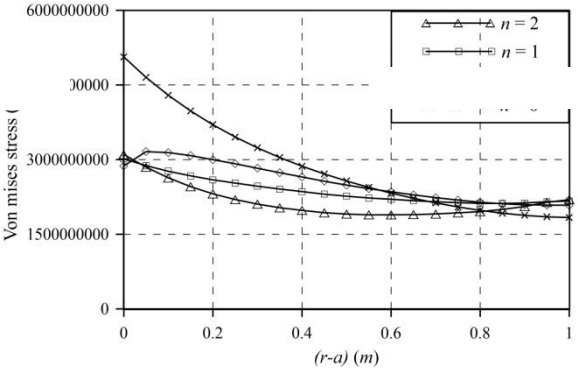


Figure 9: Distribution of von mises stress

B. Numerical results:

Figure 4 to Figure 9 shows the distribution of radial displacement, radial strain, circumferential strain, radial stress, circumferential stress and von mises stress respectively for different values of grading parameter *n*. Shells have same geometric parameters as above and rotating with an angular velocity of 500 *rad/s*, subjected to unit internal pressure that is 1 *GPa*. Material is modeled by Mori-Tanaka scheme for which properties used are as [9]: E_m = 70 *GPa*, E_c = 151 *GPa*, ρ_m = 2700 kg/m^3, ρ_c = 5700 kg/m^3, B_m = 58.333 *GPa*, B_c = 128.333 *GPa*, G_m = 26.9231 *GPa*, G_c = 58.0769 *GPa* and v = 0.3, where *B* and *G* denotes bulk modulus and shear modulus.

It is observed that radial deformation and radial as well as circumferential strains are maximum at inner radius and minimum at outer radius for all cases. n = 0 indicates unit volume fraction of the outer material means a homogeneous shell. Shell having n = 0 (homogeneous ceramic shell) has the minimum radial deformation and strains while shell having n = 2 has maximum radial deformation and strains. It is observed that radial deformation increases with an increase in grading index *n*. Increasing *n* means volume fraction of the outer material (ceramic) is decreasing and inner material (metal) is increasing, which increases radial deformation and strains. Radial strain is completely compressive and circumferential strain is completely tensile in nature. Radial stress is unit at inner radius (equal to applied pressure) and zero at outer radius for all cases, which confirms the boundary conditions applied. It can be seen from the Figure 7 that radial stress is both tensile

and compressive. Tensile nature of radial stress increases and compressive nature decreases with increasing value of grading parameter *n.* Homogeneous ceramic shell has minimum compressive stress while FGM shell having $n = 2$ has minimum tensile stress. Circumferential stress and von mises stress both have same distribution pattern. Both are completely tensile and decreases with increasing value of *n.* Homogeneous ceramic shell has maximum circumferential and von mises stress and FGM shell having grading index $n = 2$ has the least circumferential and von mises stress.

By comparing all the stresses it can be seen that von mises stress is maximum for all values of grading index *n.* Therefore von mises stress should be taken as stress criteria to design rotating spherical pressure vessel. Further it is observed that FGM shell having grading parameter $n = 2$ has the minimum von mises stress, therefore it is suggested to use this FGM for rotating spherical pressure vessel.

CONCLUSION :

The present work proposes a study of rotating FGM spherical pressure vessel using finite element method based on principle of stationary total potential. Functionally gradation of the material properties is achieved by Mori-Tanaka scheme. Material properties of aluminum metal and zirconia ceramic are used and metal-ceramic FGM is considered. The effect of gradation of material properties on stress, strain and deformation state is investigated and presented in the form of graph for different values of grading parameter. The results obtained are found to be in good agreement with established reports. It is observed that there is a significant reduction in stresses and deformation behavior of the FGM shell as compared to homogeneous shell. It is suggested that metal-ceramic FGM shell having $n = 2$ is most efficient for the purpose of rotating spherical pressure vessel among all other FGMs investigated.

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